Sigma model effective action for strong localization.

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Abstract

Bose gas in a random external field is considered. The sigma model like effective action both for weak and strong random fields compared with the interaction between particles is derived by averaging over the random field and integration over the quantum fluctuations in the framework of the Keldysh-Schwinger technique for irreversible processes. Using this effective action the properties of the system in strongly fluctuating random field are analyzed.

I. INTRODUCTION

The problem of transition between localized and extended states of many particle systems attracts attention of condensed matter theorists and experimentalists for a long time. The theory of non interacting electrons in the random external field in one and two dimensions predicts localization of particles for arbitrarily weak disorder [1], [2], [3]. In three dimensions the state of non interacting electrons is localized for sufficiently strong disorder [4], [5], and the state is extended for the disorder less than a critical edge. For negative energy with absolute value much larger than the mobility edge, the system is strongly localized and its density of states is described by so called "tail" of density of states [5], [6], [7], [8]. The common point of view is that the repulsive interaction between electrons suppresses localization. So far this problem was mainly considered in the case of week disorder [12], [13]. However the problem of investigation of transition between localized and extended states caused by interaction between particles in three dimension is of great importance and interest, but it cannot be solved in the framework of the weak disorder.

The problem of localization in a random field was investigated for the case of the Fermi statistics of particles. In this case if the Fermi energy is large enough the interaction between particles can be neglected, as it was done in [1], [2], [3], [9], [10], [11]. The analysis of the problem when both random field and the interaction between particles are involved, is the sufficiently more complicated.

In this paper we consider the case of Bose statistics of particles, although, the technique developed here, in general points, can be applied for the Fermi case too. For Bose system in random field the interaction between particles plays fundamental role and cannot be neglected. This is because the nonzero energy of the Bose system without random potential is defined by the interaction between particles, and exactly this interaction prevents the fall of the system to the very bottom of the fluctuating random potential wells. Our goal is development of a technique for the description of the transition from the localized state to the delocalized superfluid state. The experimental and theoretical investigations of this transition in the systems of different nature are of an interest for a long time [28] - [36], [21], [22], [24], [25], [23], [26], [27]. In the works [22] - [23] the Bose gas in a small random external field was considered and corrections to the temperature of the superfluid transition and to the superfluid density were calculated using perturbation theory in random field. In

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the recent works [26], [27] the case of the strong random field, or small density of Bose gas, were considered qualitatively. The technique developed here enables to consider both strong and weak random fields compared to the strength of interaction between the Bose particles. The idea to average over the random field at the first step of calculations if the Keldysh-Schwinger technique [16], [15] was already exploited previously [17], [18]. We develop this idea and obtain the effective action in the form of a sigma model, which describes both the cases of the weak and strong random field. In this work we mainly focus on the application of the obtained effective action to the case of the strong random field.

II.   AVERAGED GENERATING FUNCTIONAL

The Keldysh-Schwinger technique [15], [16], along with supersymmetry and replica technique, is known as an effective in the problems of localization in a random static potential [17], [18]. In this technique absence of denominator in the representation of generating functional makes it easy to take an average over the Gaussian random static external potential $V(\overrightarrow{r})$, provided some natural assumptions are made, like absence of interactions for asymptotic in-states which are turned on adiabatically. The generating functional for the system in random static potential $V(\overrightarrow{r})$ is

$$Z[J,V] = \int D\phi D\overline{\phi} \exp\{iS[V] + iS_J\} \tag{1}$$

with the action

$$S[V] = S_0 + \Delta S_0[V] + S_{int} \tag{2}$$

and interaction with the sources S_J. Here S_0 is the free particle action

$$S_0 = \oint dt d^d r \left\{ \overline{\phi} \left[i\partial_t - \xi\left(\widehat{\overrightarrow{p}}\right) \right] \phi \right\}, \tag{3}$$

$\Delta S_0[V]$ is the interaction of the particles with a random external field

$$\Delta S_0[V] = -\oint dt d^d r \left\{ \overline{\phi} V \phi \right\}, \tag{4}$$

and S_{int} and S_J are the interaction between the particles and their interaction with the external source fields, which are auxiliary and put to zero at the end of calculations.

$$S_{int} = -\frac{1}{2} g \oint dt d^d r \left\{ \overline{\phi}(t, \overrightarrow{r}) \phi(t, \overrightarrow{r}) \overline{\phi}(t, \overrightarrow{r}) \phi(t, \overrightarrow{r}) \right\}. \tag{5}$$

$$S_J = \oint dt d^d r \left[\overline{J}\phi + \overline{\phi}J \right] \qquad (6)$$

In (3)-(6) the time t integration contour is the standard Keldysh-Schwinger double time contour with the return, going from $t = -\infty$ to $t = +\infty$ (shifted up) and back to $t = -\infty$ (shifted down) in the complex t plane. The complex particle fields ϕ, $\overline{\phi}$ and their sources J, $\overline{J}$ are functions of the time and space coordinates $t, \overrightarrow{r}$. At the moment we don't fix the statistics of the particles, but in what follows we concentrate on the Bose particles case. g is coupling constant, and the function $\xi\left(\widehat{\overrightarrow{p}}\right)$ is the excitation spectrum of the ideal gas, $\xi\left(\widehat{\overrightarrow{p}}\right) = \frac{\overrightarrow{p}^2}{2m} - \mu$, where the momentum operator is $\widehat{\overrightarrow{p}} = -i\overrightarrow{\nabla}$, and μ is the chemical potential. We choose the system of units in which the Plank constant $\hbar$ and the mass of particles m are equal to unity $\hbar = m = 1$.

The probability of the static random field distribution is supposed to be Gaussian with the correlator of white-noise type

$$< V\left(\overrightarrow{r}\right) V\left(\overrightarrow{r}'\right) >= (1/K_0)\, \delta\left(\overrightarrow{r} - \overrightarrow{r}'\right)$$

with some constant K_0:

$$P\left[V\right] = \frac{\exp\left\{-\frac{1}{2}K_0 \int d^d r\, (V\left(\overrightarrow{r}\right))^2\right\}}{\int DV \exp\left\{-\frac{1}{2}K_0 \int d^d r d^d r'\, (V\left(\overrightarrow{r}\right))^2\right\}} \qquad (7)$$

As we said, the absence of denominator in (1) makes it possible to integrate over the random potential V in the averaged generating functional:

$$< Z\left[J,V\right] >_V = \int D\phi D\overline{\phi} DV \exp\left\{iS\left[V\right] + iS_J\right\} P\left[V\right] = \int D\phi D\overline{\phi} \exp\left\{iS\right\} \qquad (8)$$

where

$$S = S_0 + S_{int} + S_{av} + S_J \qquad (9)$$

The parts of the action S_0, S_{int}, S_J are given by equations (3), (5), (6), and new additional interaction term S_{av} generated by random field averaging, has the form

$$S_{av} = i\frac{1}{2K_0} \int d^d r \left[\oint dt \overline{\phi}\left(t, \overrightarrow{r}\right)\phi\left(t, \overrightarrow{r}\right)\right]\left[\oint dt' \overline{\phi}\left(t', \overrightarrow{r}\right)\phi\left(t', \overrightarrow{r}\right)\right] \qquad (10)$$

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It is convenient to rewrite the action S in the so called three-angle representation [16], [19], and introduce the fields Ψ and ψ instead of the fields ϕ_1 and ϕ_2 , where 1, 2 are the indexes of the upper and lower brunches of the time contour respectively:

$$\Psi = \frac{\phi_1 + \phi_2}{2}; \qquad \psi = \phi_1 - \phi_2 \qquad (11)$$
$$J = \frac{J_1 + J_2}{2}; \qquad j = j_1 - j_2$$

In these variables the parts of the action take the form

$$S_0 = \int dt d^d r \left\{ \overline{\psi} \left(\widehat{G}_{0R}^{-1} \right) \Psi + \overline{\Psi} \left(\widehat{G}_{0A}^{-1} \right) \psi \right\} \qquad (12)$$

$$S_{int} = -g \int dt d^d r \left\{ \left(\overline{\Psi}\psi + \overline{\psi}\Psi \right) \left(\overline{\Psi}\Psi + \frac{1}{4}\overline{\psi}\psi \right) \right\} \qquad (13)$$

$$S_{av} = \frac{i}{2K_0} \int d^d r \left\{ \int dt \left[\overline{\Psi}\psi + \overline{\psi}\Psi \right] \right\} \left\{ \int dt' \left[\overline{\Psi}\psi + \overline{\psi}\Psi \right] \right\} \qquad (14)$$

$$S_J = \int dt d^d r \left\{ \left(\overline{J}\psi + \overline{\psi}J \right) + \left(\overline{j}\Psi + \overline{\Psi}j \right) \right\} \qquad (15)$$

where the inverse retarded and advanced free field Green functions $\widehat{G}_{0R}$, $\widehat{G}_{0A}$ in frequency representation are

$$\left(\widehat{G}_{0R}^{-1} \right) = \omega + \mu + \frac{1}{2m}\overrightarrow{\nabla}^2 + i\gamma \qquad (16)$$
$$\left(\widehat{G}_{0A}^{-1} \right) = \omega + \mu + \frac{1}{2m}\overrightarrow{\nabla}^2 - i\gamma$$

with infinitesimal positive γ. In time representation ω should be replaced by $i\partial_t$. In the three-angle representation asymptotic of the fields ψ, $\overline{\psi}$ at $t \to \infty$ satisfy the condition $\psi(t \to \infty, \overrightarrow{r}) = \overline{\psi}(t \to \infty, \overrightarrow{r}) = 0$, and the fields Ψ, $\overline{\Psi}$ are non zero at $t \to \infty$ [17]. Note that the classical fields, i.e. the fields which in functional integral technique correspond not to operators but to c-number values, have $\psi = \phi_1 - \phi_2$ equal to zero at any time moment. The fields ψ, $\overline{\psi}$ describe quantum fluctuations of the quantum system, and Ψ, $\overline{\Psi}$ can be considered as the classical components of the fields [17], [18]. In the case of large occupation of some degrees of freedom the quantum fluctuations ψ, $\overline{\psi}$ corresponding to these degrees of

freedom give small corrections compared to the classical components Ψ, $\overline{\Psi}$, and in principle can be taken into account in the lower orders of perturbation theory. If the system has Bose condensate it is described by the fields Ψ, $\overline{\Psi}$.

III. INTEGRATION OVER QUANTUM FLUCTUATIONS.

Application of the Keldysh-Schwinger technique for localization problem in Fermi systems in weak random field [18] was based on the construction of so called Q-Lagrangian [9], like for example in replica or supersymmetry technique [9]-[14]. Correctness of application of this technique to strong localization problem is doubtful. The main goal of this paper is to consider the properties of many-particle system in strong random field. Here we consider Bose gas of small density in strong random field. We suppose $ng^3 << 1$, where n is the averaged density of the gas, g is related to the scattering length [20], and the fluctuations of random field are assumed to be larger or of the order of $\mu_r = ng$.

The novelty of our approach is that we start from integration over the quantum fields ψ, $\overline{\psi}$, getting an effective action for the classical fields Ψ, $\overline{\Psi}$. In order to integrate over ψ, $\overline{\psi}$ we introduce new fields η, $\overline{\eta}$ by

$$\psi\left(t, \overrightarrow{r}\right) = \frac{1}{\left(\overline{\Psi}\left(t, \overrightarrow{r}\right)\Psi\left(t, \overrightarrow{r}\right)\right)}\Psi\left(t, \overrightarrow{r}\right)\eta\left(t, \overrightarrow{r}\right) \tag{17}$$

$$\overline{\psi}\left(t, \overrightarrow{r}\right) = \overline{\eta}\left(t, \overrightarrow{r}\right)\overline{\Psi}\left(t, \overrightarrow{r}\right)\frac{1}{\left(\overline{\Psi}\left(t, \overrightarrow{r}\right)\Psi\left(t, \overrightarrow{r}\right)\right)}$$

We preserved the form of equations which will be valid for both statistics of the fields ψ, Ψ, although our goal in this paper is, as we said, the bosonic case. In this context the above equations are written in the way which avoids a division by a Grassmann variable in the fermionic case. Here and below the form of equation (17) is also convenient in the case of additional degrees of freedom for the field Ψ, like spin. The action S_{av} is Gaussian in η, $\overline{\eta}$. Our strategy will be to integrate over the fields η, $\overline{\eta}$ and write the effective action in terms of the chiral fields. Without the interaction term S_{int} this integration is exact both for Fermi and Bose cases. The effective action takes the form of a sigma model. In this paper the effective action will be obtained for the Bose gas with the repulsive interaction between particles, but the technique used here works in fermionic case as well.

The Jacobian of transformations (17) is

$$\det\left(\frac{\delta\psi, \delta\overline{\psi}}{\delta\eta, \delta\overline{\eta}}\right) = \prod_{t,\overrightarrow{r}}\frac{1}{\left(\overline{\Psi}\Psi\right)} \tag{18}$$

In terms of new variables $\eta, \overline{\eta}$ the action S_{av} takes the form

$$S_{av} = \frac{i}{2K_0}\int d^d r \int dt \int dt'\, (\overline{\eta}(t, \overrightarrow{r}) + \eta(t, \overrightarrow{r}))\,(\overline{\eta}(t', \overrightarrow{r}) + \eta(t', \overrightarrow{r})) \tag{19}$$

This is the crucial point of the calculation: there are two factorized time integrations in the last equation.

For future convenience we rewrite the action, introducing new (chiral) fields $\overline{e}, e, \overline{f}, f$ instead of the classical fields $\overline{\Psi}, \Psi$:

$$e = \frac{1}{\sqrt{\overline{\Psi}\Psi}}\Psi; \qquad \overline{e} = \overline{\Psi}\frac{1}{\sqrt{\overline{\Psi}\Psi}} \tag{20}$$

$$f = \exp\left(\frac{1}{2}\lambda\right); \qquad \overline{f} = \exp\left(-\frac{1}{2}\lambda\right) \tag{21}$$

$$\lambda = \ln\left(\frac{\overline{\Psi}\Psi}{n_c^{(0)}}\right) \tag{22}$$

The constant $n_c^{(0)}$, independent of $\overrightarrow{r}$, is introduced to make the argument of logarithm dimensionless. The chiral fields satisfy the following constraints

$$\overline{e}e = 1, \quad \overline{f}f = 1 \tag{23}$$

At this stage we will neglect the highest order terms of the quantum fluctuations ψ in S_{int} (13) proportional to ψ^3. It is quite well understood [15], [16], [19], [18] that these terms are responsible for relaxation processes induced by interaction between the particles [20]. These interaction responsible terms can be neglected if the density of the system or the interaction between particles are small $ng^3 << 1$. Lets emphasis that this approximation, nevertheless, give the possibility to consider random field fluctuations of any strength exactly. They are essential if the kinetics of relaxation to an equilibrium state is considered. In this paper we suppose that the system is at an equilibrium state and there are no relaxation processes.

With this assumption one can see that in terms of new fields (20)-(22) the free and the interaction parts of the action (12),(13) combine to

$$S_0 + S_{int} = \int dt d^d r \left\{ \overline{\eta} L + \overline{L} \eta \right\} \tag{24}$$

where

$$L\left(t, \overrightarrow{r}\right) = \overline{f}\left(i\partial_t f\right) + \overline{e}\left(i\partial_t e\right) + \mu + \frac{1}{2m}\overline{e}\left(\overrightarrow{\nabla}^2 e\right) + \tag{25}$$
$$+ \frac{1}{2m}\overline{f}\overrightarrow{\nabla}^2 f + \frac{1}{2m}\left(\overrightarrow{\nabla}\lambda\right)\left(\overline{e}\overrightarrow{\nabla}e\right) - gn_c\left(f\right)^2$$

$$\overline{L}\left(t, \overrightarrow{r}\right) = \left(i\partial_t \overline{f}\right) f + \left(-i\partial_t \overline{e}\right) e + \mu + \frac{1}{2m}\left(\overrightarrow{\nabla}^2 \overline{e}\right) e + \tag{26}$$
$$+ \frac{1}{2m}\overline{f}\overrightarrow{\nabla}^2 f + \frac{1}{2m}\left(\overrightarrow{\nabla}\lambda\right)\left(\left(\overrightarrow{\nabla}\overline{e}\right) e\right) - gn_c^{(0)}\left(f\right)^2$$

$$L + \overline{L} = \overline{e}\left(i\partial_t e\right) + \left(-i\partial_t \overline{e}\right) e + 2\mu - \frac{1}{m}\left(\overrightarrow{\nabla}\overline{e}\right)\left(\overrightarrow{\nabla}e\right) + \tag{27}$$
$$+ \frac{1}{m}\overline{f}\overrightarrow{\nabla}^2 f - 2gn_c^{(0)}\left(f\right)^2$$

The constant $n_c^{(0)}$ with dimension of density is arbitrary and actually drops out from the equation (27) because of the gradient operator and Eq. (21). We will fix its choice below.

Eq. (27) was obtained from Eqs. (25, 26) using the constraint (23) on the fields $e, \overline{e}$. The remaining part of the action S_J can be written as

$$S_J = \int dt d^d r \left\{ (n_c^{(0)})^{-1/2}\left(\overline{J}e\eta + \overline{\eta e}J\right)\overline{f} + (n_c^{(0)})^{1/2}f\left(\overline{j}e + \overline{e}j\right) \right\} \tag{28}$$

Equations (20) can be written in polar coordinates ρ, φ for complex fields $\Psi, \overline{\Psi}$:

$$\Psi = \rho \exp\left(i\varphi\right) = \rho e; \quad \overline{\Psi} = \rho \exp\left(-i\varphi\right) = \rho \overline{e}; \tag{29}$$
$$\rho = \sqrt{\overline{\Psi}\Psi}$$

Note that the functional integration measure changes under the transformation $\Psi, \overline{\Psi} \to \lambda, \varphi$ as

$$\prod_{t,\overrightarrow{r}} \frac{1}{(\overline{\Psi}\Psi)} D\Psi D\overline{\Psi} D\eta D\overline{\eta} = \prod_{t,\overrightarrow{r}} D\lambda D\varphi D\eta D\overline{\eta}$$

If we split out the zero frequency mode (time independent) part η_0, L_0 of η, L

$$\eta(t, \overrightarrow{r}) = \frac{1}{T}\eta_0(\overrightarrow{r}) + \delta\eta(t, \overrightarrow{r}); \qquad \overline{\eta}(t, \overrightarrow{r}) = \frac{1}{T}\overline{\eta}_0(\overrightarrow{r}) + \delta\overline{\eta}(t, \overrightarrow{r}) \qquad (30)$$

$$L(t, \overrightarrow{r}) = L_0(\overrightarrow{r}) + \delta L(t, \overrightarrow{r}); \qquad \overline{L}(t, \overrightarrow{r}) = \overline{L}_0(\overrightarrow{r}) + \delta\overline{L}(t, \overrightarrow{r})$$

with the total time evolution $T \to \infty$, the action S_{av} (19) becomes just

$$S_{av} = \frac{2i}{K_0} \int d^d r \, (\eta_0(\overrightarrow{r}))^2 \qquad (31)$$

whilst the other terms of the action take the form

$$S_0 + S_{int} = \int d^d r \left(\frac{\int dt \, (L_0 + \overline{L}_0)}{T} \right) \eta_0 + \int d^d r dt \left(\delta\overline{\eta}\delta L + \delta\overline{L}\delta\eta \right) \qquad (32)$$

Since the last term in (32) is the only one where $\delta\eta, \delta\overline{\eta}$ appears, functional integration over them gives additional constraint $\delta L = \delta\overline{L} = 0$. It means that only time independent (zero frequency time Fourier components of the) functions $L, \overline{L}$ contribute to the functional integral. However, in general it doesn't mean that fields $e, \overline{e}, f, \overline{f}$ are time independent. L, $\overline{L}$ can be taken at any time moment, e.g. at $t = \infty$. With this choice we will omit the time argument for the functions $e, \overline{e}, f, \overline{f}$ below.

The first two terms of $L + \overline{L}$ Eq. (27) in the first summand can be transformed to a renormalization of the chemical potential: In terms of polar variables (29) they are transformed into

$$\frac{1}{T} \int d^d r dt \left[\overline{e}(i\partial_t e) + (-i\partial_t \overline{e}) e \right] \eta_0 = -2 \int d^d r \frac{\varphi(\infty, \overrightarrow{r}) - \varphi(-\infty, \overrightarrow{r})}{T} \eta_0(\overrightarrow{r})$$

The difference of phases $\Delta\varphi(\overrightarrow{r}) = \varphi(\infty, \overrightarrow{r}) - \varphi(-\infty, \overrightarrow{r})$ can be written as $\Delta\varphi(\overrightarrow{r}) = \delta\varphi(\overrightarrow{r}) + 2\pi n$, where $0 \leq \delta\varphi(\overrightarrow{r}) < 2\pi$ and n is an integer number, which should not depend on the space coordinate $\overrightarrow{r}$. The changes of $\Delta\varphi(\overrightarrow{r})$ by discontinuous jumps means a divergence of its gradient, and as a consequence to zero contribution to the functional integral. Thus, the dependence on the constant integer number n can be included into the renormalization of the chemical potential μ, and the term $\delta\varphi/T \tilde{} 2\pi/T$ can be neglected for large T.

Integration over η_0 in the generating functional $< Z[J, V] >_V$ finally gives

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$$< Z\left[J, V\right] >_V = \int DeDf \exp\left\{-S_{eff} + iS_j\right\} \tag{33}$$

where

$$S_{eff} = \frac{K_0}{8} \int d^d r \left[L\left(\overrightarrow{r}\right) + \overline{L}\left(\overrightarrow{r}\right)\right]^2 \tag{34}$$

and S_j is also expressed in terms of the fields e, f. The effective action S_{eff} can be written in the form

$$S_{eff} = \frac{1}{2^{1+d/2}} \left(\frac{|\mu|}{E_0}\right)^{2-d/2} \int d^d R L_{eff}^2(\overrightarrow{R}) \tag{35}$$

where E_0 is the Larkin energy, which characterizes the scale of the fluctuating random potential V.

$$E_0 = \frac{1}{(K_0)^{2/(4-d)}}$$

and $L_{eff}(\overrightarrow{R})$ is dimensionless representation of the function $\frac{1}{2}\left(L\left(\overrightarrow{r}\right) + \overline{L}\left(\overrightarrow{r}\right)\right)$:

$$L_{eff}(\overrightarrow{R}) = \pm 1 - \left(\overrightarrow{\nabla}_R \overline{e}\right)\left(\overrightarrow{\nabla}_R e\right) + \overline{f}\left(\overrightarrow{\nabla}_R\right)^2 f - \frac{g}{|\mu|} n_c^{(0)}\left(f\right)^2 \tag{36}$$

The sign of ± 1 here is the sign of the chemical potential μ, and $\overrightarrow{R}$ is the dimensionless coordinate variable

$$\overrightarrow{R} = \overrightarrow{r}\sqrt{2|\mu|} \tag{37}$$

and the gradient operator is $\overrightarrow{\nabla}_R = \frac{\partial}{\partial \overrightarrow{R}}$. The dimensionless interaction constant $\alpha = \frac{g}{|\mu|} n_c^{(0)}$ can be fixed to unity if we choose $n_c^{(0)}$ as

$$n_c^{(0)} = \frac{|\mu|}{g} \tag{38}$$

and the Lagrangian L_{eff} takes the form

$$L_{eff}(\overrightarrow{R}) = \pm 1 - \left(\overrightarrow{\nabla}_R \overline{e}\right)\left(\overrightarrow{\nabla}_R e\right) + \overline{f}\left(\overrightarrow{\nabla}_R\right)^2 f - \left(f\right)^2 \tag{39}$$

Note that the effective action Eq. (35) is derived without any assumptions about weakness of disorder and can be used for the description of the transition from strong localization to delocalization and superfluidity.

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IV.   CALCULATION OF CHEMICAL POTENTIAL.

In this section, as an application of the derived effective potential, we calculate the density of particles in three spacial dimensions ($d = 3$). The correlator $< \overline{\Psi}\left(\overrightarrow{r},t\right)\Psi\left(\overrightarrow{r},t\right) >$ defining the density of the particles, can be written in the form of functional integral

$$< \overline{\Psi}\left(\overrightarrow{r},t\right)\Psi\left(\overrightarrow{r},t\right) >= \int DeD\overline{e}DfD\overline{f}\left(f\left(\overrightarrow{r}\right)\right)^2 \exp\left\{-S_{eff}\right\} \tag{40}$$

We consider the case of the large negative chemical potential $\mu < 0$, $\mid \mu \mid >> E_0$. The total number of particles N is defined by this correlator

$$N = \int d^3 r < \overline{\Psi}\left(\overrightarrow{r},t\right)\Psi\left(\overrightarrow{r},t\right) > \tag{41}$$

and below we calculate the chemical potential μ as a function of the average particles density $n = N/\Omega$, where Ω is the total space volume of the system.

It is clear that only e, f field configurations with non diverging L_{eff} will contribute to the functional integral (40). In particular, it means that $L_{eff}\left(\overrightarrow{R}\right)$ should tend to zero rapidly enough for $\overrightarrow{R} \rightarrow \infty$. Thus, for large $\overrightarrow{R}$, we have the equation

$$-\left(\overrightarrow{\nabla}_R\overline{e}\right)\left(\overrightarrow{\nabla}_R e\right) + \overline{f}\left(\overrightarrow{\nabla}_R\right)^2 f - 1 - (f)^2 = 0$$

Solution of this equation defines the asymptotic $\overrightarrow{R} \rightarrow \infty$ of an instanton. In the whole region of distances R the instanton is defined by extremum of the action S_{eff}. Lets find solutions with $e = const$. Then the equation takes the form

$$\overline{f}\left(\overrightarrow{\nabla}_R\right)^2 f - 1 - (f)^2 = 0$$

Using the constraint $\overline{f}f = 1$, we obtain

$$\left(\overrightarrow{\nabla}_R\right)^2 f - f - (f)^3 = 0 \tag{42}$$

We will look for solution in the class of spherically symmetric functions of the form $f = \exp(\frac{1}{2}\lambda)$. Then Laplacian is

$$\left(\overrightarrow{\nabla}_R\right)^2 = \frac{d^2}{dR^2} + \frac{2}{R}\frac{d}{dR} \tag{43}$$

Thus, for large $R \rightarrow \infty$ Eq. (42) can be written as

$$\frac{d^2}{dR^2}f - f - (f)^3 = 0 \tag{44}$$

Moreover, we will look for solutions with the asymptotic

$$f(R) = \exp(-R) \quad \text{as} \quad R \to \infty \tag{45}$$

which means that the third term in Eq. (44) can be neglected at $R \to \infty$. The center of the instanton is an arbitrary parameter $\overrightarrow{r}_0$, or its dimensionless analog $\overrightarrow{R}_0 = \sqrt{2|\mu|}\overrightarrow{r}_0$.

$$f\left(\overrightarrow{R}\right) = \exp\left(-|\overrightarrow{R} - \overrightarrow{R}_0|\right) \quad \text{as} \quad \overrightarrow{R} \to \infty \tag{46}$$

and the integration over $\overrightarrow{R}_0$ is the integration over the zero mode. All instantons which differ only by parameter $\overrightarrow{R}_0$ give the same contribution to the functional integral, due to the translational invariance of the effective action.

So we get in quasi classical limit for the correlator

$$< \overline{\Psi}\left(\overrightarrow{r}, t\right) \Psi\left(\overrightarrow{r}, t\right) > \tag{47}$$

$$= n_c^{(0)} \int d^3 R_0 \left(f^{inst}\left(\overrightarrow{R}\right)\right)^2 \exp\left\{-\frac{1}{2^{5/2}} \left(\frac{|\mu|}{E_0}\right)^{1/2} \int d^3 R \left(L^{inst}\right)^2\right\}$$

where

$$L^{inst} = \overline{f}^{inst} \left(\overrightarrow{\nabla}_R\right)^2 f^{inst} - 1 - \left(f^{inst}\right)^2$$

One can look for the instanton configuration in the form

$$f^{inst}\left(\overrightarrow{R}\right) = \frac{\exp\left(-|\overrightarrow{R} - \overrightarrow{R}_0|\right)}{\sqrt{|\overrightarrow{R} - \overrightarrow{R}_0|^2 + \varkappa^2}} \tag{48}$$

where the parameter $\varkappa$ should be found from the minimization of the action S_{eff} (35) calculated on this configuration. We found that $\varkappa = \left(\frac{3}{4}\right)^{2/3}$. The effective action S_{eff} for this value of $\varkappa$ is

$$S_{eff} = \left(\frac{|\mu|}{E_0}\right)^{1/2} A$$

where

$$A = \frac{1}{2^{5/2}} \int d^3 R \left(L^{inst} \right)^2 \sim 4 \tag{49}$$

Finally we get the following estimation for the correlator

$$< \overline{\Psi} \left(\overrightarrow{r}, t \right) \Psi \left(\overrightarrow{r}, t \right) >= n_c^{(0)} \int d^3 R_0 \left(f^{inst} \left(\overrightarrow{R} \right) \right)^2 \exp \left\{ - \left(\frac{|\mu|}{E_0} \right)^{1/2} A \right\} \tag{50}$$

Hence the total number of particles

$$N = n_c^{(0)} \int d^3 r \int D\overrightarrow{R}_0 \left(f^{inst} \left(\overrightarrow{R} \right) \right)^2 \exp \left\{ - \left(\frac{|\mu|}{E_0} \right)^{1/2} A \right\} = \tag{51}$$

$$= \Omega n_c^{(0)} \exp \left\{ - \left(\frac{|\mu|}{E_0} \right)^{1/2} A \right\}$$

Thus using Eqs. (51) and (38) we get the equation for the chemical potential, which is

$$\frac{ng}{|\mu|} = \exp \left\{ - \left(\frac{|\mu|}{E_0} \right)^{1/2} A \right\} \tag{52}$$

Strictly speaking, this equation is valid for the large values of the chemical potential so far as the saddle point calculation of the functional integral (40) is valid for $|\mu| >> E_0$. However, we think that, as one can guess from the form of the effective action (35), the equation of this form is valid for $|\mu| \sim ng \sim E_0$ too. In this case the effective action is dimensionless and does not depend on any parameter. The main contribution to the functional integral (40) in this case provided by the field f with the asymptotic (46), as as it is for instantons in the saddle point approximation. This contribution should be of the order of unity, so the estimation for the chemical potential is $|\mu| \sim ng \sim E_0$. Eq. (52) has a solution for the densities smaller than the critical density $n < n_c$, and does not have a solution for $n > n_c$, where $n_c \sim \frac{E_0}{g}$. Formally, solution of Eq. (52), if exists, has two branches (see Fig.1 for illustration). But the situation is different from the first order phase transitions, since only one of them is stable. The second solution has negative compressibility $\frac{\partial \mu}{\partial n} < 0$, and should be rejected. In a sense, the situation recalls a bifurcation point rather a first order phase transition. The stable solution of (52) with respect to μ as a function of n, in the region $|\mu| >> E_0$ looks like

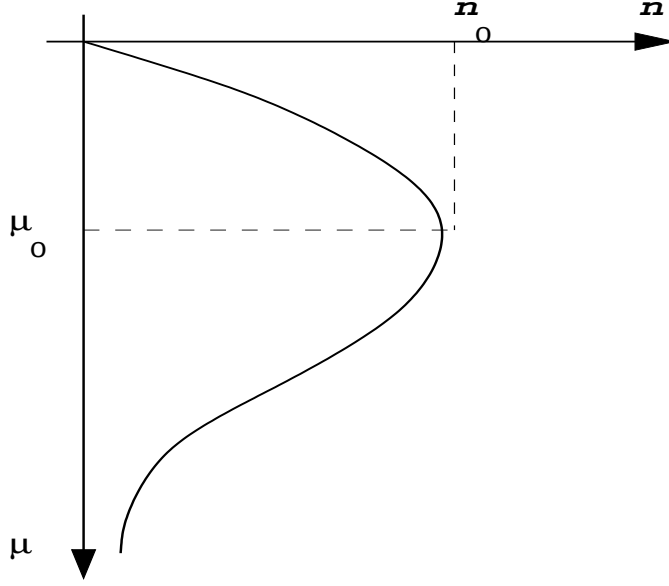$$\mu = -E_0 \ln^2 \left(\frac{E_0}{ng} \right) \tag{53}$$

Fig.1 Chemical potential as a function of density n.

It coincides with the expression for the chemical potential obtained recently in [26], [27]. The value of the chemical potential $|\mu|$ for densities $n \lesssim n_c$ can be estimated as $\mu \sim \mu_c$, where $\mu_c \sim -E_0$. When the density n becomes more than n_c there is no solution of Eq. (52), which means that for $n > n_c$ the chemical potential will change the sign to positive and the system becomes delocalized and superfluid.

Note that the exponential in Eq. (51) is typical for the tail of the density of states in 3D for the negative energy of particles in a random potential.

In order to understand long distance correlation properties of the system in the regime $n < n_c$ consider the correlator $< \overline{\Psi}\left(\overrightarrow{r}_1, t\right) \Psi\left(\overrightarrow{r}_2, t\right) >$ for $\mid \overrightarrow{r}_1 - \overrightarrow{r}_2 \mid \to \infty$. The functional integral in the form of the new variables is

$$< \overline{\Psi}\left(\overrightarrow{r}_1\right) \Psi\left(\overrightarrow{r}_2\right) >= \int DfDe \exp\left\{-S_{eff}\right\} f\left(\overrightarrow{r}_1\right) f\left(\overrightarrow{r}_2\right) \overline{e}\left(\overrightarrow{r}_1\right) e\left(\overrightarrow{r}_2\right)$$

In the saddle point approximation the configurations of fields $\left(f\left(\overrightarrow{r}_1\right), e\left(\overrightarrow{r}_1\right)\right)$, $\left(f\left(\overrightarrow{r}_2\right), e\left(\overrightarrow{r}_2\right)\right)$, when $\mid \overrightarrow{r}_1 - \overrightarrow{r}_2 \mid \to \infty$, are described by the instantons localized near far distant points $\overrightarrow{r}_1$ and $\overrightarrow{r}_2$, respectively, and the correlator $< \overline{\Psi}\left(\overrightarrow{r}_1\right) \Psi\left(\overrightarrow{r}_2\right) >$ is exponetially small, i.e. the system has no long range order for densities $n < n_c$. Note that the correlator $< \overline{e}\left(\overrightarrow{r}_1\right) e\left(\overrightarrow{r}_2\right) >$ is equal to zero for $\mid \overrightarrow{r}_1 - \overrightarrow{r}_2 \mid \to \infty$ due to the independence of the phases of the different instantons.

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V. DISCUSSION

The effective action for many-particle system in random potential is derived in the assumption of weak interaction between particles but without restrictions on the strength of the random field fluctuations. The approach we develop is different from the usual Q-Lagrangian approach used in the theory of weak localization. The order parameter which could characterize the transition from localized to delocalized state has another structure and characterized by the "classical" field Ψ. We obtain the effective action by the integration over the random external field at the first stage, following by integration over the quantum fluctuations described by the field ψ. The approximation we use for the integration over the field ψ is correct in quasi classical case as well as for small density of the system or weak interaction between particles. In this paper we consider the properties of Bose gas in random external potential with the Gaussian correlator for the random field fluctuations in the case of the negative chemical potential, i.e., in the region of the strong localization. The obtained effective action has instantons which are localized in space and give the finite value for the effective action. The contribution of these instantons to the functional integral (40) defines the equation for the chemical potential as a function of the averaged density. The solution of this equation exists only if the averaged density is smaller than the critical value $n_c \sim \frac{E_0}{g}$. The chemical potential for the densities $n \sim n_c$ is of the order $\mu \sim \mu_c = -E_0$.

We have shown that the system has no long range order for the densities $n < n_c$, whilst the chemical potential becoms positive, and the strong localized state transfers to the delocalized superfluid state.

One of the main important problems remaining for future is a more careful semiclassical investigation of instantonic solutions found here for different space dimensions. It would be also interesting to apply the developed method to the regime $\mu \sim \mu_c$, where it could be possible to see the phase transition, probably of the first order, when μ changes the sign.

Another problem where the described path integral Keldysh-Schwinger technique can be applied, is fermionic particles in a random potential. Here higher order powers of electron-electron interaction cannot be neglected, and the full action should be treated non perturbatively. It seams reasonable to concentrate on the 1+1 dimensional space case for this problem and try to analyze a relation of the found chiral Lagrangian to solvable sigma model cases.

VI. ACKNOWLEDGMENTS

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